

On phase transition signal in inelastic collision

J.Manjavidze

JINR, Dubna, Russia¹

Abstract

The paper is devoted to retrieval of the first order phase transition signal in the inelastic collisions. The primary intent is to show that the experimentally observable signal exist iff the multiplicity is sufficiently large. We discuss corresponding phenomenology from the point of view of experiment.

I

The question of possibility to observe first order phase transition in the hadron and heavy ion collisions is discussed widely at present time [1]. The aim of paper is discuss the phenomenology of that problem.

The first order phase transition in statistics appears in the result of creation of local critical fluctuations (e.g. bubbles of vapor if the (liquid→gas) transition is considered), in contrast to the second order phase transition where the whole system undergo the transition. The dimension of such fluctuations increase, if they are under-critical, and the whole system in result undergo the transition.

It is hard to imagine that exactly such picture appears in the hadron or ion inelastic collisions. The point is that in the event-by-event measurement the dimension of bubbles, *if they exist*, may be both smaller or larger critical one. Therefore, in the best case, we observe the mixture of two-phase medium and the question how one may increase on experiment the weight of events with under-critical bubbles is the first problem. It is evident that the density, usually used in statistics, can not be introduced as the "order parameter" since only the result of particles *production* process is observed.

In the paper [2] the "chemical potential", $\mu(n, s)$, was offered as the "order parameter". It is the work which is necessary for creation of one particle and it was shown that if the role of under-critical bubbles dominate then $\mu(n, s)$ must decrease with number of produced particles n . In another words, it was offered to define the boiling, i.e. creation of under-critical bubbles, through *intensity* of the process of particles production (evaporation).

This is the main idea why the very high multiplicity (VHM) processes were considered. It is not important what additional criterium is used searching first order phase transition. In any case we must consider the VHM domain to have intensive production of particles. In addition, the kinetic degrees of freedom must be suppressed in the VHM region.

Therefore, the special attention will be given to the VHM processes. Then exist approximation [2]:

$$\mu(n, s) \simeq -\frac{T(n, s)}{n} \ln \sigma_n(s) \quad (1)$$

in this multiplicity region. Here T is the mean energy, including mass, of produced particles, i.e. T is associated with temperature, and σ_n is the normalized to unite multiple production cross section which can be considered in the VHM region as the "partition function" of the *equilibrium* system. The equilibrium condition will be defined later, see inequality (17), definition (18) and [3] where the detailed explanation was given. See also the footnote 5.

¹ On leave in absence: Andronikashvili Institute of Physics, Tbilisi, Georgia

Continuing the analogy with thermodynamics one can say that $(-T \ln \sigma_n)/n$ is the Gibbs free energy per one particle. Then μ can be interpreted as the "chemical potential" measured with help of *observed* free particles².

The definition (1) is quiet general. It can be used both for hadron-hadron and ion-ion collisions, both for low and high energies, both for "boiling" media of coloured partons and colorless hadrons. Definition (1) is model free and operates only with "external" directly measurable parameters. The single indispensable condition: we work in the VHM region.

We will discuss in the paper the chance of experimental measurement of $\mu(n, s)$ defined by (1), what kind uncertainties hides it from *experimental* point of view noting the the cross sections in VHM domain are small. The correction to (1) are not essential in the VHM region but nevertheless the field-theoretical definition of chemical potential in using Wigner functions formalism [4] will be published, see also [3].

II.

It is necessary to remind main steps toward (1) to understand hidden phenomenological uncertainties. The starting point [2] was the generating function

$$\rho(z, s) = \sum_{n=0}^{\infty} z^n \sigma_n(s), \quad \rho(1, s) = 1, \quad \sigma_n = 0 \text{ at } n > n_{max} = \sqrt{s}/m, \quad (2)$$

where m is the hadron mass. One may use inverse Mellin transformation:

$$\sigma_n(s) = \frac{1}{2\pi i} \oint \frac{dz}{z^{n+1}} \rho(z, s) \quad (3)$$

to find σ_n if $\rho(z, s)$ is known. One may calculate integral (3) by saddle point method. The equation (of state):

$$n = z \frac{\partial}{\partial z} \ln \rho(z, s) \quad (4)$$

defines mostly essential value $z = z(n, s)$. Therefore, only

$$z < z_{max} = z(n_{max}, s) \quad (5)$$

have the physical meaning.

One may write $\rho(z, s)$ in the form:

$$\rho(z, s) = \exp \left\{ \sum_{l=0}^{\infty} z^l b_l(s) \right\}, \quad (6)$$

where the "Mayer group coefficient" b_l can be expressed through correlators $c_k(s)$:

$$b_l(s) = \sum_{k=l}^{\infty} \frac{(-1)^{(k-l)}}{l!(k-l)!} c_k(s).$$

Let us assume now that we have Poisson distribution, i.e. if in the sum:

$$\ln \rho(z, s) = \sum_k \frac{(z-1)^k}{k!} c_k(s) \quad (7)$$

one may leave first term, then it is easy to see that

$$z(n, s) = n/c_1(s), \quad c_1(s) \equiv \bar{n}(s), \quad (8)$$

²Notice that one may consider n as the multiplicity in the experimentally observable range of phase space.

are essential and in the VHM region:

$$\ln \sigma_n(s) = -n \ln \frac{n}{c_1(s)} (1 + O(1/\ln n)) = -n \ln z(n, s) (1 + O(1/\ln n)). \quad (9)$$

Therefore, in considered case with $c_k = 0$, $k > 1$, exist following asymptotic estimation for $n \gg 1$:

$$\ln \sigma_n \simeq -n \ln z(n, s), \quad (10)$$

i.e. σ_n is defined in VHM region mainly by the solution of Eq.(4) and the correction can not change this conclusion. It will be shown that that kind estimation is hold for arbitrary asymptotics of σ_n . The definition (1) based on this observation.

If we understand σ_n as the "partition function" in the VHM region then z is the *activity* usually introduced in statistical physics. Correspondingly the chemical potential μ is defined trough z :

$$\mu = T \ln z. \quad (11)$$

Combining this definition with estimation (10) we define σ_n through μ . But, *if* this estimation does not depend from asymptotics of σ_n over n , i.e. if it has general meaning, then it can be used for definition of $\mu(n, s)$ through $\sigma_n(s)$ and $T(n, s)$ at $n \gg 1$. Just this formal idea is realized in (1): it can be shown in Sec.III that (1) is correct at the asymptotical value of n .

III.

Now we will make the important step. To put in a good order our intuition it is useful to consider $\rho(z, s)$ as the *nontrivial* function of z . In statistical physics the thermodynamical limit is considered for this purpose. In our case the finiteness of energy \sqrt{s} and of the hadron mass m put obstacles on this way since the system of produced particles necessarily belongs to the energy-momentum surface³. But we can continue *theoretically* σ_n to the range $n > n_{max}$ and consider $\rho(z, s)$ as the nontrivial function of z . This step hides the assumption that nothing new appear at $n > n_{max}$, i.e. the VHM interval $\bar{n} \ll n < n_{max}$ is sufficiently wide to represent main physical processes.

Let us consider the analog generating function which has the first $n < n_{max}$ coefficient of expansion over z equal to σ_n and higher coefficients for $n \geq n_{max}$ are deduced from continuation of theoretical value of σ_n to $n \geq n_{max}$. Then the inverse Mellin transformation (3) gives a good estimation of σ_n through this generating function if the fluctuations near $z(n, s)$ are Gaussian or, it is the same, if

$$\frac{|2n - z^3 \partial^3 \ln \rho(z, s) / \partial z^3|}{|n + z^2 \partial^2 \ln \rho(z, s) / \partial z^2|^{3/2}} \Big|_{z=z(n, s)} \ll 1. \quad (12)$$

Notice that if the estimation (10) is generally rightful then one can easily find that l.h.s. of (12) is $\sim 1/n^{1/2}$. Therefore, one may consider $\rho(z, s)$ as the nontrivial function of z considering $z(n, s) < z_{max}$ if $\bar{n} \ll n < n_{max}$.

Then it is easily deduce that the asymptotics of $\sigma_n(s)$ is defined by the leftmost singularity, z_c , of function $\rho(z, s)$ since, as it follows from Eq.(4), the singularity "attracts" the solution $z(n, s)$ *in the VHM region*. In result we may classify asymptotics of σ_n in the VHM region if (12) is hold.

Our problem is reduced to the definition of possible location of leftmost singularity of $\rho(z, s)$ over $z > 0$ ⁴. It must be stressed that the character of singularity is not important

³It must be noted that the canonical thermodynamic system belongs to the energy-momentum shell because of the energy exchange, i.e. interaction, with thermostat. The width of the shell is defined by the temperature. But in particle physics there is no thermostat and the physical system completely belongs to the energy momentum surface.

⁴The singularities in complex z plane will not be considered since they lead only to oscillations in multiplicity distribution.

for definition of $\mu(n, s)$ in the VHM region at least with $O(1/\ln n)$ accuracy. One may consider only three possibility at $n \rightarrow \infty$: (I) $z(n, s) \rightarrow z_c = 1$; (II) $z(n, s) \rightarrow z_c$, $1 < z_c < \infty$; (III) $z(n, s) \rightarrow z_c = \infty$. The structure of complex z plane is much more complicate but for our purpose the above described picture is sufficient.

Correspondingly one may consider only three type of asymptotics in the VHM region: (I) $\sigma_n > O(e^{-n})$; (II) $\sigma_n = O(e^{-n})$. Such asymptotics is typical for hard processes with large transverse momenta, like for jets [3]; (III) $\sigma_n < O(e^{-n})$. That asymptotic behavior is typical for multiperipheral-like kinematics, where the longitudinal momenta of produced particles are noticeably higher than the transverse ones [3].

Therefore the case (I) is the best candidate for phase transition since in this case the cross sections are comparatively large in the VHM region, i.e. particles "intensively" produced in that case. Notice that if (I) is not realized in nature then the (II) kind processes would dominate in the VHM region.

Let us consider now the estimation (1). It follows from (3) that, up to the preexponential factor,

$$\ln \sigma(n, s) \approx -n \ln z(n, s) + \ln \rho(z(n, s), s). \quad (13)$$

We want to show that, in a wide range of n from VHM region,

$$n \ln z(n, s) \sim \ln \rho(z(n, s), s). \quad (14)$$

Let us consider now the mostly characteristic examples.

(I) *Singularity at $z = 1$.* The physical meaning of singularity at $z = 1$ may be illustrated by the droplet model [6]. The Mayer's group coefficient, see (6), for cluster from l particle is $b_l(\beta) \sim \exp\{-\beta \tau l^{(d-1)/d}\}$, where $\tau l^{(d-1)/d}$, $l \gg 1$, is the surface tension energy, d is the dimension. Therefore, if $d > 1$ the series over l in (6) diverges at $z = 1$.

This case was considered in [2] in details. In the used lattice gas approximation $\ln z(n) \sim n^{-5}$ and $\ln \sigma_n \approx -n^{-4} = -n \ln z(n)(1 + O(1/n))$. Notice that the simplest droplet model predicts unphysical asymptotics: $\sigma_n \rightarrow \text{const}$ in the VHM region.

(II) *Singularity at $1 < z_c < \infty$.* Let us consider one jet contribution: $\ln \rho(z, s) = -\gamma \ln(1 - \bar{n}_j(s)(z - 1))$. In this case $z(n, s) = z_c(1 - \gamma/n)$, $n \gg \gamma$, and $\ln \sigma_n = -n \ln z(n, s)(1 + O(\ln n/n))$.

(III) *Singularity at $z = \infty$.* For k Pomeron exchange: $\ln \rho(z, s) = c_k(s)(z - 1)^k$. In this case $z(n) = (n/kc_k)^{1/k} \gg 1$ and $\ln \sigma_n \approx -n \ln z(n)(1 + O(1/\ln n))$.

One can conclude:

(i) The definition (1) in the VHM region is rightful since the correction falls down with n . On this stage we can give only the qualitative estimation of corrections. Nevertheless (1) gives the correct n dependence in the VHM region.

(ii) Activity $z(n, s)$ tends to z_c from the right in the case (I) and from the left if we have the case (II) or (III).

(iii) The accuracy of estimation of the chemical potential (1) increase from (III) to (I).

IV.

The temperature T is the next problem. The temperature is introduced usually using Kubo-Martin-Schwinger (KMS) periodic boundary conditions. But this way assumes from the very beginning that the system (a) is equilibrium and (b) is surrounded by thermostat through which the temperature is determined. The first condition (a) we take as the simplification which gives the equilibrium state.

The second one (b) is the problem since there is no thermostat in particle physics. For this reason we introduce the temperature as the Lagrange multiplier $\beta = 1/T$ of energy conservation law [3]. In such approach the condition that the system is in

equilibrium with thermostat replaced by the condition that the fluctuations in vicinity of β are Gaussian.

The interesting for us $\rho(z, s)$ we define through inverse Laplace transform of $\rho(z, \beta)$:

$$\rho(z, s) = \int \frac{d\beta}{2\pi i \sqrt{s}} e^{\beta \sqrt{s}} \rho(z, \beta). \quad (15)$$

It is known that if the interaction radii is finite, i.e. the hadron mass is finite, then the equation (of state):

$$\sqrt{s} = -\frac{\partial}{\partial \beta} \rho(z, \beta) \quad (16)$$

have real positive solution $\beta(n, s)$ at $z = z(n, s)$. We will assume that the fluctuations near $\beta(n, s)$ are Gaussian. This means that the inequality [3]:

$$\left| \frac{|\partial^3 \ln \rho(z, \beta) / \partial \beta^3|}{|\partial^2 \ln \rho(z, \beta) / \partial \beta^2|^{3/2}} \right|_{z=z(n, s), \beta=\beta(n, s)} \ll 1 \quad (17)$$

is satisfied. Therefore, we prepare the formalism to find "thermodynamic" description of the processes of particle production assuming that this S -matrix condition of equilibrium (17) is hold⁵.

I want to underline that our thermal equilibrium condition (17) have absolute meaning: if it is not satisfied then $\beta(n, s)$ loses every sense since the expansion in vicinity of $\beta(n, s)$ leads to the asymptotic series. In this case only the dynamical description of S -matrix can be used.

It is not hard to see [3] that

$$\frac{\partial^l}{\partial \beta^l} \ln R(z, \beta) |_{z=z(n, s), \beta=\beta(n, s)} = < \prod_{i=1}^l (\eta_i - < \eta >) >_{n, s} \quad (18)$$

is the l -point energy correlator, where $< \dots >_{n, s}$ means averaging over all events with given multiplicity and energy. Therefore (17) means "relaxation of l -point correlations", $l > 2$, measured in units of the dispersion of energy fluctuations, $l = 2$. One can note here the difference of our definition of thermal equilibrium from thermodynamical one [5].

V.

We may conclude that:

(i) The definition of chemical potential (1) was discussed. This important observable can be measured on the experiment directly. Chemical potential, $\mu(n, s)$, must decrease in the VHM region if the first order phase transition occur, case (I), and it rise in opposite case, see (II) and (III), see Sec.III.

(ii) We are forced to assume that the energy and the multiplicity are sufficiently large, i.e. the experimental value $z^{exp}(n, s)$ is sufficiently close to $z_c = 1$. In opposite case the leading leftmost singularity over z would not be "seen" on experiment and the production processes constitutes from the complicated mixture of subprocesses.

(iii) The cross section σ_n falls down rapidly with n and for this reason the VHM events are hardly observable. One may avoid this problem considering the finite energy heavy ion collisions as the most candidates of processes described by methods of thermodynamics and z_c is easier "reachable" in this case.

⁵Introduction of $\beta(n, s)$ allows to describe the system of large number of degrees of freedom in terms of single parameter $\beta(n, s)$, i.e. it is nothing but the useful trick. It is no way for this reason to identify entirely $1/\beta(n, s)$ with thermodynamic temperature where it has the self-contained physical sense. Nevertheless path-integral representation of $\rho(\beta, z)$ defined from S -matrix coincides with Feynman-Kac representation of grand partition function [3] if (17) is hold. It must be noted also that the energy spectrum of produced particle in this case have Boltzmann form, $e^{-\beta \eta}$.

(iv) One can define $z(n, s)$ also directly from Eq.(4):

$$n = z \frac{\partial}{\partial z} \ln \sum_n z^n \sigma_n^{exp}(s), \quad (19)$$

using experimental values $\sigma_n^{exp}(s)$. But comparing (19) with definition (10),

$$n \ln z \simeq - \ln \sigma_n^{exp}(s), \quad (20)$$

it seems that last one gives more definite value of $z^{exp}(n, s)$ than the "integral" equation (19) especially since the statistical errors are large in the VHM region and the theoretical correction to Eq.(20) are small, $\sim 1/n$.

Summarizing the results we conclude: *if the energy is sufficiently large, i.e. if z_{max} is sufficiently close to $z_c = 1$, if the multiplicity is sufficiently large, so that (17) is satisfied and $z(n, s)$ can be sufficiently close to z_c , then one may have confident answer on the question: observable or not the first order phase transition in hadron/ion collisions. The heavy ion collisions are favorable to observe the phase transition.*

Acknowledgements.

I would like to thank participants of 7-th International Workshop on the "Very High Multiplicity Physics" (JINR, Dubna) for stimulating discussions. I am grateful to V.Priezzhev, A.Sissakian, A.Sorin and V.Kekelidze for valuable attention.

References

- [1] BNL Report, *Hunting the Quark Gluon Plasma*, BNL-73847-2005; C.Alt et al., The NA-49 Collaboration, nucl-ex/0710.0118; M.Creutz, Phys. Rev., D15 (1977) 1128; M.Gazdzicki and M.I.Gorenstein, Acta Physica Polonica, B 30, 2705, (1999)
- [2] J.Manjavidze and A.Sissakian, *Proc. VHM Physics Workshops*, (World Scient., 2008)
- [3] J.Manjavidze and A.Sissakian, Phys. Rep., 346 (2001) 1, hep-ph/0105245
- [4] J.Manjavidze, Phys.Part.Nucl. 30 (1999) 49, hep-ph/9802318
- [5] N.N.Bogolyubov, *Studies in Statistical Mechanics*, (North-Holland Publ. Co., Amsterdam, 1962)
- [6] T.D.Lee and C.N.Yang, Phys.Rev., 87 (1952) 404, 410; J.S.Langer, Ann.Phys., 41 (1967) 108